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Softly broken lepton numbers and maximal neutrino mixing

Walter Grimus
Institut für Theoretische Physik, Universität Wien
Boltzmanngasse 5, A–1090 Wien, Austria
E-mail: grimus@doppler.thp.univie.ac.at

Luís Lavoura
Universidade Técnica de Lisboa
Centro de Física das Interacções Fundamentais
Instituto Superior Técnico, P–1049-001 Lisboa, Portugal
E-mail: balio@cfif.ist.utl.pt

Abstract: We consider lepton mixing in an extension of the Standard Model with three right-handed neutrino singlets. We require that the three lepton numbers L_e, L_μ, and L_τ be separately conserved in the Yukawa couplings, and we assume that they are *softly* broken only by the Majorana mass matrix M_R of the neutrino singlets. In this framework, where lepton-number breaking occurs at a scale much higher than the electroweak scale, deviations from family-lepton-number conservation are calculable and finite, and lepton mixing stems exclusively from M_R. We then show that a discrete symmetry exists such that, in the lepton mixing matrix U, maximal atmospheric neutrino mixing together with $U_{e3} = 0$ can be obtained *naturally*. Alternatively, if one assumes that there are two different scales in M_R and that the lepton number $\bar{L} = L_e - L_\mu - L_\tau$ is conserved in between them, then maximal solar neutrino mixing follows naturally. If both the discrete symmetry and intermediate $\bar{L}$ conservation are introduced, bimaximal mixing is achieved.

Keywords: Global Symmetries, Solar and Atmospheric Neutrinos, Beyond Standard Model, Neutrino Physics.

Contents

1. Introduction

At present the positive results of some experimental searches for neutrino oscillations provide the only glimpse of physics beyond the Standard Model (SM). In particular, the results of the atmospheric and solar neutrino experiments have a natural explanation in terms of neutrino oscillations [1] and, therefore, in terms of non-zero light-neutrino masses m_k ($k = 1, 2, 3$) and lepton mixing. Atmospheric neutrino results [2] are well fitted by a mass-squared difference $\Delta m^2_{\mathrm{atm}} = |m_3^2 - m_2^2| \simeq 3 \times 10^{-3}\,\mathrm{eV}^2$ and a mixing angle $\psi \sim 45°$. The situation for solar neutrinos is not that clear-cut: several viable explanations of the solar-neutrino deficit seem to exist (for recent analyses see [3]–[6]; for the latest experimental results of Super-Kamiokande see [7]). The large-mixing-angle (LMA) MSW [8] solution has $\Delta m^2_\odot = |m_2^2 - m_1^2| \simeq 3 \times 10^{-5}\,\mathrm{eV}^2$ and a large mixing angle θ; the small-mixing-angle (SMA) MSW solution features $\Delta m^2_\odot \simeq 5 \times 10^{-6}\,\mathrm{eV}^2$ and $\tan^2\theta \simeq 6 \times 10^{-3}$; finally, the LOW solution has $\Delta m^2_\odot \simeq 10^{-7}\,\mathrm{eV}^2$ and a large mixing angle, possibly reaching 45°. (All these numbers have been taken from [4].)

The problem of neutrino masses and lepton mixing can be separated into two questions. The first one concerns mechanisms for achieving neutrino masses which are much smaller than the charged-lepton masses; the seesaw mechanism [9] figures as a prominent solution to this problem and will be adopted in the present paper. The second question concerns the specific features of the lepton mixing matrix U; in particular, how can an atmospheric mixing angle close to 45°, and a very small U_{e3} [10] (see also [5, 6]), be explained. This second question has been viewed from many different angles (see for instance [11]) and no preferred solutions have emerged yet. In particular, it has been observed [12] that the lepton number $\bar{L} = L_e - L_\mu - L_\tau$, which must be broken at some stage [13] to obtain $\Delta m^2_\odot \neq 0$, leads to maximal *solar* mixing — see also [14] and the references therein. This is certainly an interesting observation, but it may be argued that explaining maximal *atmospheric* mixing should have a higher priority than explaining maximal solar mixing — since the latter is not yet firmly established by experiment. Now, it has proven difficult to obtain maximal atmospheric mixing in a *natural* fashion (i.e., protected by some symmetry), in particular because the charged-lepton mass matrix may be non-diagonal and blur the picture of mixing following from the neutrino mass matrix.

In the present paper we take up and expand the idea of using lepton numbers for enforcing desired features upon the mixing matrix U. We start from the lepton sector of the SM enlarged by three right-handed neutrino singlets with Majorana mass terms given by the mass matrix M_R. The scalar sector of our theory consists only of SU(2) doublets and we do not introduce other scalar multiplets like singlets or triplets. The main idea of our work is to impose separate conservation of the family lepton numbers L_e, L_μ, and L_τ (for a recent review on the status of lepton numbers see [15]) and to allow them to be broken only *softly*, a breaking which may occur exclusively in M_R. Thus, the family lepton numbers are broken by mass terms of dimension three at a scale much higher than the electroweak scale. Under these conditions, lepton-number-breaking processes have calculable and finite amplitudes. This framework has the advantage that lepton mixing originates solely in the neutrino Majorana mass matrix M_R; contributions to U from the charged-lepton mass matrix are naturally forbidden at tree level by the assumed conservation of the family lepton numbers in all terms of dimension four in the lagrangian. In this framework, we then show that it is possible to impose a discrete symmetry which leads to maximal atmospheric mixing and to $U_{e3} = 0$ (i.e., decoupled solar and atmospheric neutrino oscillations [16]). Alternatively, it is possible to have two different mass scales $m_R \ll \bar{m}_R$ in M_R such that for energies in between m_R and $\bar{m}_R$ the lepton number $\bar{L}$ is conserved; we then arrive at a scenario with maximal solar mixing.[1] Both scenarios may be combined to yield bimaximal neutrino mixing [17].

In section 2 we discuss the SM with family lepton numbers broken exclusively by the Majorana mass terms of right-handed neutrino singlets. We show that flavour-changing neutral interactions are not enhanced in spite of the large right-handed mass scale. In section 3 we introduce a model which naturally yields maximal atmospheric mixing, whereas in section 4 we discuss intermediate $\bar{L}$ conservation and maximal solar mixing. Both features are combined in section 5. The conclusions of this paper are found in section 6.

[1]An earlier variant of this idea, with two right-handed neutrino singlets only, was proposed in [14].

2. The framework

The framework within which we develop our models consists of the lepton sector of the SM with three families together with three right-handed neutrino singlets ν_R. We allow for an arbitrary number n_H of Higgs doublets ϕ_j ($j = 1, \ldots, n_H$) and use the notation

$$\phi_j = \begin{pmatrix} \varphi_j^+ \\ \varphi_j^0 \end{pmatrix} \qquad \text{and} \qquad \langle 0 \,|\varphi_j^0|\, 0 \rangle = \frac{v_j}{\sqrt{2}} \,. \tag{2.1}$$

We first compile various well-known formulae to fix the notation. The right-handed neutrino singlets have a Majorana mass term

$$\mathcal{L}_M = \frac{1}{2}\, \nu_R^T C^{-1} M_R^* \nu_R - \frac{1}{2}\, \bar{\nu}_R M_R C \bar{\nu}_R^T \,, \tag{2.2}$$

where M_R is symmetric. We define the left-handed neutrino singlets $\nu_L' \equiv C\bar{\nu}_R^T$, then

$$\mathcal{L}_M = \frac{1}{2}\, \nu_L'^T C^{-1} M_R \nu_L' - \frac{1}{2}\, \bar{\nu}_L' M_R^* C \bar{\nu}_L'^T . \tag{2.3}$$

The Yukawa lagrangian of the leptons is given by

$$\mathcal{L}_{\mathrm{Y}} = -\sum_{j=1}^{n_H} \left[\bar{\ell}_R \,(\, \varphi_j^-, \quad \varphi_j^{0*}\,)\, \Gamma_j + \bar{\nu}_R \,(\, \varphi_j^0, \quad -\varphi_j^+\,)\, \Delta_j \right] \begin{pmatrix} \nu_L \\ \ell_L \end{pmatrix} + \text{h.c.} \tag{2.4}$$

Defining the charged-lepton mass matrix M_ℓ and the Dirac neutrino mass matrix M_D as

$$M_\ell = \frac{1}{\sqrt{2}} \sum_j v_j^* \Gamma_j \qquad \text{and} \qquad M_D = \frac{1}{\sqrt{2}} \sum_j v_j \Delta_j \,, \tag{2.5}$$

respectively, we have from $\mathcal{L}_{\mathrm{Y}}$ the following mass terms for the charged leptons and for the neutrinos:

$$\mathcal{L}_{\mathrm{Y_{mass}}} = -\bar{\ell}_R M_\ell \ell_L + \frac{1}{2}\, \nu_L'^T C^{-1} M_D \nu_L + \frac{1}{2}\, \nu_L^T C^{-1} M_D^T \nu_L' + \text{h.c.} \tag{2.6}$$

The Dirac and Majorana mass terms for the neutrinos may be written together as

$$\frac{1}{2}\,(\, \nu_L^T, \quad \nu_L'^T\,)\, C^{-1} \mathcal{M}_{D+M} \begin{pmatrix} \nu_L \\ \nu_L' \end{pmatrix} + \text{h.c.} \qquad \text{with } \mathcal{M}_{D+M} = \begin{pmatrix} 0 & M_D^T \\ M_D & M_R \end{pmatrix} . \tag{2.7}$$

We assume that there are (at least) two mass scales in the theory: the electroweak scale, i.e., the one of the vacuum expectation values v_j, and the scale of the Majorana mass terms, i.e., the one of the eigenvalues of $\sqrt{M_R^* M_R}$. The latter scale is assumed to be much higher than the electroweak scale. Then, the neutrino fields participating in the weak interaction, ν_L, have effective Majorana mass terms given by

$$\mathcal{L}_m = \frac{1}{2}\, \nu_L^T C^{-1} \mathcal{M}_\nu \nu_L + \text{h.c.} \,, \tag{2.8}$$

with the (approximate) 3×3 seesaw mass matrix [9]

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D \,. \tag{2.9}$$

The 6×6 Majorana mass matrix of eq. (2.7) is diagonalized by the unitary matrix $\mathcal{U}$ such that

$$\mathcal{U}^T \mathcal{M}_{D+M} \mathcal{U} = \hat{m} = \mathrm{diag}\,(m_1, \ldots, m_6)\,, \tag{2.10}$$

where $m_{1,2,3}$ are seesaw-suppressed whereas $m_{4,5,6}$ are very large. It is useful to decompose $\mathcal{U}$ into two 3×6 matrices U_L and U_R in the following way:

$$\mathcal{U} = \begin{pmatrix} U_L \\ U_R^* \end{pmatrix} \simeq \begin{pmatrix} \mathbf{1} & M_D^\dagger M_R^{*\,-1} \\ -M_R^{-1} M_D & \mathbf{1} \end{pmatrix} \begin{pmatrix} V & 0 \\ 0 & W^* \end{pmatrix}, \tag{2.11}$$

where, in the second part of the equation, $\mathcal{U}$ has been given to leading order in the inverse of the high scale [18]. At that order, the 3×3 unitary matrices V and W are defined by

$$V^T \mathcal{M}_\nu V = \mathrm{diag}\,(m_1, m_2, m_3) \qquad \text{and} \qquad W^\dagger M_R W^* = \mathrm{diag}\,(m_4, m_5, m_6)\,. \tag{2.12}$$

The foundation of our models lies in the assumption of the conservation of *all* three lepton numbers L_α ($\alpha = e, \mu, \tau$) in the Yukawa couplings and, in general, in all terms of the lagrangian with dimension four. This means that the three right-handed charged leptons ℓ_R may from the start receive generation labels — we call them e_R, μ_R, and τ_R. Similarly, each of the three right-handed neutrinos ν_R carries one unit of the corresponding lepton number L_α, and is accordingly named ν_{eR}, $\nu_{\mu R}$, or $\nu_{\tau R}$. Finally, the three left-handed lepton doublets $(\nu_L,\ \ell_L)^T$ will be called D_e, D_μ, and D_τ. The Yukawa coupling matrices Γ_j and Δ_j, and the mass matrices M_ℓ and M_D, are all simultaneously diagonal. The generation lepton numbers L_α are broken explicitly *but softly* only by the dimension-three mass terms in $\mathcal{L}_M$ of eq. (2.2).[2]

As M_ℓ is diagonal, and since in our model there are three charged leptons together with six neutrinos, the 3×6 matrix U_L in eq. (2.11) is the lepton mixing matrix. The part of the mixing matrix relevant for the light neutrinos is approximately unitary and is given by V; up to a phase convention, this matrix is usually called the neutrino mixing matrix U, i.e.,

$$V = e^{i\hat{\alpha}} U e^{i\hat{\beta}}\,, \tag{2.13}$$

where the diagonal phase matrix $e^{i\hat{\alpha}}$ has no physical meaning (it may be absorbed in the phases of the charged-lepton fields) while the diagonal phase matrix $e^{i\hat{\beta}}$ contains phases which appear only in the lepton-number-violating processes typical of the Majorana character of the neutrinos.

[2] The renormalization-group evolution of the Yukawa-coupling matrices Γ_j and Δ_j does not introduce flavour-changing elements [19].

We claim that in this framework, in any process, the deviation from family-lepton-number conservation proceeds in a finite and calculable way, controlled by the elements of M_R. In particular, radiative corrections introduce flavour-changing interactions of the neutral scalars due to $\mathcal{L}_M$. At one-loop level, the logarithmic term induced by charged-Higgs exchange, which contains the leading term,[3] is given by

$$\Delta\Gamma_{j,FC} = \frac{-1}{16\sqrt{2}\,\pi^2} \sum_{i=1}^{n_H} \Gamma_i \Delta_j^\dagger U_R \ln\left(\frac{\hat{m}^2}{m^2}\right) U_R^\dagger \Delta_i\,, \tag{2.14}$$

where U_R is the 3×6 matrix introduced in eq. (2.11). The mass m is arbitrary, since only the off-diagonal terms of eq. (2.14), which are finite and m-independent, have physical significance. If we choose m to be the right-handed-neutrinos mass scale, we see that the large logarithms $\ln\left(m_{1,2,3}^2/m^2\right)$ are suppressed by small mixing angles in U_R. The flavour-changing neutral interactions are weak because they are cubic in the Yukawa coupling constants. This guarantees that our theory is viable. For instance, there is a stringent bound of order 10^{-12} on the branching ratio of $\mu^- \to e^- e^+ e^-$ [21]. With the result of eq. (2.14) we can estimate that branching ratio to be of order $Y^8/\left(16\pi^2 G_F m_H^2\right)^2$, where Y is a typical Yukawa coupling and m_H is a typical neutral-scalar mass. Taking $m_H \sim 100\,\text{GeV}$ and $Y \sim 10^{-3}$–10^{-2} (Y should be of order of a charged-lepton mass divided by the Fermi scale), we obtain a branching ratio of 10^{-18} or smaller, quite safe when compared to the experimental bound.

Charged-scalar exchange also induces the radiative decay $\mu^- \to e^-\gamma$, which has an experimental upper bound on its branching ratio of order 10^{-11} [21]. As before, the transition amplitude for this decay has both light and heavy neutrinos in the loop. There is also a chirality flip and, therefore, a mass insertion. The calculation of $\mu^- \to e^-\gamma$ in the present theory is not similar to the one in the context of the Zee model [22], but rather to the ones in the context of supersymmetric models — see for instance [23]. The branching ratio of $\mu^- \to e^-\gamma$ is suppressed in our model not only by the fine-structure constant α and a product of four Yukawa couplings, but also due to a GIM mechanism [24]. A crude estimate of an upper bound on this branching ratio is given by $\alpha Y^4/\left(\pi G_F^2 m_R^4\right)$, where m_R denotes the scale of the Majorana mass terms of the right-handed neutrinos. This one-loop estimate is far below the present experimental bound. A detailed account of this decay and of other flavour-changing decays in the context of our framework will be presented in a forthcoming paper [20].

3. Maximal atmospheric mixing

Within the theory described in the previous section it is possible to implement maximal atmospheric mixing naturally. A key to this possibility is the conservation of

[3] This term originates in the vertex correction with the charged scalars in the loop. The flavour-changing interactions will be studied in detail in a forthcoming paper [20].

the lepton numbers L_e, L_μ, and L_τ in the Yukawa interactions, making the charged-lepton mass matrix diagonal; we have to worry only about the form of M_R and M_D. We consider $n_H = 3$ and introduce two Z_2 symmetries:

$$\begin{array}{lllll} Z_2^{(1)}: & \nu_{\mu R} \leftrightarrow \nu_{\tau R}\,, & D_\mu \leftrightarrow D_\tau\,, & \mu_R \leftrightarrow \tau_R\,, & \phi_3 \to -\phi_3\,; \\ Z_2^{(2)}: & \mu_R \to -\mu_R\,, & \tau_R \to -\tau_R\,, & \phi_2 \to -\phi_2\,, & \phi_3 \to -\phi_3\,. \end{array} \tag{3.1}$$

Fields not appearing in these equations transform trivially. Let us motivate these choices. Because of $Z_2^{(1)}$ we have

$$\left(M_R\right)_{e\mu} = \left(M_R\right)_{e\tau} \quad \text{and} \quad \left(M_R\right)_{\mu\mu} = \left(M_R\right)_{\tau\tau}\,. \tag{3.2}$$

As ϕ_2 and ϕ_3 change sign under $Z_2^{(2)}$, only ϕ_1 has Yukawa couplings to the neutrinos:

$$\mathcal{L}_{\mathrm{Y}}(\nu_R) = -\frac{\sqrt{2}}{v_1}\left(\varphi_1^0,\, -\varphi_1^+\right)\left[a\bar{\nu}_{eR}D_e + b\left(\bar{\nu}_{\mu R}D_\mu + \bar{\nu}_{\tau R}D_\tau\right)\right] + \text{h.c.}\,, \tag{3.3}$$

cf. eq. (2.4). Thus,

$$M_D = \operatorname{diag}\left(a, b, b\right) \tag{3.4}$$

has identical μ and τ entries, once again because of $Z_2^{(1)}$. As a consequence, the light-neutrino Majorana mass matrix has the same structure as M_R:

$$\mathcal{M}_\nu = \begin{pmatrix} x & y & y \\ y & z & w \\ y & w & z \end{pmatrix}. \tag{3.5}$$

Maximal atmospheric neutrino mixing and $U_{e3} = 0$ immediately follow from this structure of $\mathcal{M}_\nu$.[4] Using an adequate phase convention, cf. eq. (2.13), to transit from the unitary matrix V which diagonalizes $\mathcal{M}_\nu$ to the neutrino mixing matrix U, we obtain

$$U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ \sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} & -1/\sqrt{2} \\ \sin\theta/\sqrt{2} & -\cos\theta/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}. \tag{3.6}$$

Indeed, as the charged-lepton mass matrix is diagonal within our general framework, the matrix U is already the neutrino mixing matrix. The third column of U in eq. (3.6) is an eigenvector of the mass matrix $\mathcal{M}_\nu$ of eq. (3.5). The form of the other two columns of U follows from the form of the third column. At tree level the atmospheric mixing angle is exactly 45°, whereas the solar mixing angle θ is free. Without loss of generality we may assume $m_1 < m_2$ and $0^\circ < \theta < 90^\circ$.

Up to now a single Higgs doublet ϕ_1 was needed. Coming to the charged-lepton masses, we obviously have to break $Z_2^{(1)}$ in order to avoid $m_\mu = m_\tau$. This is achieved

[4]This structure of $\mathcal{M}_\nu$ in the basis where the charged-lepton mass matrix is diagonal has been suggested by several authors, e.g. [25, 26, 27]. We stress that in our case this structure results from a symmetry, i.e., we have a *model* and not just a *texture* for $\mathcal{M}_\nu$.

by introducing two more doublets ϕ_2 and ϕ_3, one of them being even and the other one being odd under $Z_2^{(1)}$. These doublets must not couple to ν_R because we want to avoid destruction of the form of M_D in eq. (3.4); this is the rationale for introducing the symmetry $Z_2^{(2)}$. We obtain the following Yukawa couplings to the charged leptons:

$$\begin{aligned}\mathcal{L}_{\rm Y}(\ell_R) = &-\frac{\sqrt{2}m_e}{v_1^*}\left(\varphi_1^-, \quad \varphi_1^{0*}\right)\bar{e}_R D_e - \sqrt{2}d\left(\varphi_2^-, \quad \varphi_2^{0*}\right)\left(\bar{\mu}_R D_\mu + \bar{\tau}_R D_\tau\right) - \\ &- \sqrt{2}d'\left(\varphi_3^-, \quad \varphi_3^{0*}\right)\left(\bar{\mu}_R D_\mu - \bar{\tau}_R D_\tau\right) + \text{h.c.}\,, \end{aligned} \tag{3.7}$$

yielding

$$\begin{aligned} m_\mu &= |d v_2^* + d' v_3^*|\,, \\ m_\tau &= |d v_2^* - d' v_3^*|\,. \end{aligned} \tag{3.8}$$

This allows for $m_\mu \neq m_\tau$, with some finetuning in order to obtain $m_\tau \gg m_\mu$. Such a finetuning is, anyway, needed even within the SM.

We need to check that the Higgs potential is such that it does not possess any U(1) symmetries apart from the weak hypercharge U$(1)_Y$. In particular, we easily see that all terms of the form $(\phi_i^\dagger \phi_j)^2$ are allowed by both $Z_2^{(1)}$ and $Z_2^{(2)}$; therefore, there are no unwanted Goldstone bosons and all the physical-scalar masses can be made sufficiently heavy, of the order of the weak scale. Notice that both $Z_2^{(1)}$ and $Z_2^{(2)}$ are broken spontaneously, but at tree level this breaking is not felt in the neutrino sector.

In our model the seesaw mechanism is operative. If we assume a and b to be of order $m_\nu^D \sim 0.1$–$1\,\text{GeV}$ (where m_ν^D is a typical charged-lepton mass), and if m_R^2 is the order of magnitude of the eigenvalues of $M_R^* M_R$, then we obtain the order-of-magnitude estimate $\sqrt{\Delta m^2_{\rm atm}} \sim (m_\nu^D)^2/m_R$, which gives $m_R \sim 10^8$–$10^{10}\,\text{GeV}$.

Let us deal a bit longer with $\mathcal{M}_\nu$ of eq. (3.5). The neutrino masses are given by

$$m_3 = |z - w| \tag{3.9}$$

and

$$m_{1,2}^2 = \frac{1}{2}\left[|x|^2 + 4|y|^2 + |z+w|^2 \mp \sqrt{\left(|x|^2 + 4|y|^2 + |z+w|^2\right)^2 - 4\left|x(z+w) - 2y^2\right|^2}\right]. \tag{3.10}$$

The solar mixing angle is expressed as

$$\tan 2\theta = 2\sqrt{2}\,\frac{|x^* y + y^*(z+w)|}{|z+w|^2 - |x|^2}\,. \tag{3.11}$$

Notice in particular that the only physical phase in $\mathcal{M}_\nu$ is

$$\beta = \arg x + \arg(z+w) - 2\arg y\,. \tag{3.12}$$

Indeed, we have

$$\tan 2\theta = 2\sqrt{2}\,|y|\,\frac{\left|e^{i\beta}|z+w| + |x|\right|}{|z+w|^2 - |x|^2} \tag{3.13}$$

and

$$\Delta m^2_\odot = \sqrt{\left(|z+w|^2 - |x|^2\right)^2 + 8\,|y|^2\,|e^{i\beta}\,|z+w| + |x||^2}\,. \tag{3.14}$$

Combining these two equations leads to [26]

$$\Delta m^2_\odot\, \cos 2\theta = |z+w|^2 - |x|^2\,, \tag{3.15}$$

where we have taken into account that, by definition, θ lies in the first quadrant. This model does not allow to relate $\Delta m^2_\odot$ with $\Delta m^2_{\rm atm}$, since m_3 is independent from $m_{1,2}$.

Given any values of $\Delta m^2_\odot$, $\Delta m^2_{\rm atm}$, and θ, they can be reproduced within the present model. First we use eq. (3.15) to express $|z+w|$, plug this expression into eq. (3.11) and arrive at

$$\sin 2\theta = 2\sqrt{2}\,\frac{|x||y|}{\Delta m^2_\odot}\left|1 + e^{i\beta}\sqrt{1+\Delta m^2_\odot \cos 2\theta/|x|^2}\right|. \tag{3.16}$$

Obviously, by choosing $|x|$ and $|y|$ we can achieve any desired value of θ, and by choosing $|z-w| = m_3$ we reproduce the experimental value of the atmospheric mass-squared difference.

Let us now address the question of whether one can still fit any values of $\Delta m^2_\odot$, $\Delta m^2_{\rm atm}$, and θ, when $|x|$, $|y|$, $|z+w|$, and $|z-w|$ are all assumed to be of order $\sqrt{\Delta m^2_{\rm atm}}$. A glance at eq. (3.16) shows that this is impossible for $|1+e^{i\beta}| \gg \Delta m^2_\odot/\Delta m^2_{\rm atm}$. However, if we assume for simplicity that $e^{i\beta} = -1$, then we have

$$\tan 2\theta \simeq \sqrt{2}\,\frac{|y|}{|x|} \tag{3.17}$$

and, in general, we have a large solar neutrino mixing. Naturally, with our order-of-magnitude assumption on the absolute values, finetuning is required to make the solar mass-squared difference small. This finetuning is expressed, e.g., by eq. (3.15) [26].

At this stage it is appropriate to ask whether there is any reason for having $e^{i\beta} = -1$. Since we need a real phase factor, we explore the effect of CP invariance:

$$CP: \quad D_\alpha \to \gamma^0 C \bar{D}^T_\alpha\,, \quad \ell_R \to \gamma^0 C \bar{\ell}^T_R\,, \quad \nu_R \to \gamma^0 C \bar{\nu}^T_R\,, \quad \phi_j \to \phi^*_j\,. \tag{3.18}$$

This CP symmetry leads to real Yukawa-coupling matrices Γ_j and Δ_j, while M_R is imaginary. With eq. (2.9) and the definition of β it is easy to check that[5]

$$\begin{aligned} e^{i\beta} &= \mathrm{sign}\left\{(M_R)_{ee}\left[(M_R)_{\mu\mu} + (M_R)_{\mu\tau}\right] \Big/ (M_R)^2_{e\mu}\right\} \\ &= -\mathrm{sign}\left\{(M_R)_{ee}\left[(M_R)_{\mu\mu} + (M_R)_{\mu\tau}\right]\right\}. \end{aligned} \tag{3.19}$$

[5]Note that we have the requirement $(M_R)^2_{\mu\mu} \neq (M_R)^2_{\mu\tau}$. Indeed, for $(M_R)_{\mu\mu} = (M_R)_{\mu\tau}$ the matrix M_R is singular, while for $(M_R)_{\mu\mu} = -(M_R)_{\mu\tau}$ we have $x = 0$ and the phase β is not defined.

JHEP07(2001)045

It is thus natural to have $\beta = \pi$ if we invoke CP invariance. In this case, with all the matrix elements of $\mathcal{M}_\nu$ of order $\sqrt{\Delta m^2_{\rm atm}}$, the only remaining finetuning which is required is to make $|z+w| - |x|$ sufficiently small, see eq. (3.14).

4. Maximal solar mixing

In this section we dispense with the $Z_2^{(1)}$ and $Z_2^{(2)}$ symmetries of the previous section. Our present aim is to show that, if one assumes the three individual lepton numbers L_e, L_μ, and L_τ to be broken down at some high scale $\bar{m}_R$ to their linear combination $\bar{L} = L_e - L_\mu - L_\tau$, and one further assumes that $\bar{L}$ only gets broken at a much lower scale m_R [14], then approximate maximal solar neutrino mixing follows. The crucial point is that the assumption of $\bar{L}$ conservation in between the two high scales $\bar{m}_R$ and m_R is natural in the technical sense, since $\bar{L}$ is a symmetry, and therefore maximal solar mixing is a natural option in the context of our framework.

Let us define the small dimensionless parameter $\epsilon = m_R/\bar{m}_R \ll 1$. The mass matrix M_R then has the form

$$M_R = \begin{pmatrix} u & p/\epsilon & q/\epsilon \\ p/\epsilon & r & t \\ q/\epsilon & t & s \end{pmatrix} \tag{4.1}$$

where u, p, q, r, s, and t are assumed to be all of order of magnitude m_R. Indeed, intermediate $\bar{L}$ conservation implies that the (e, μ) and (e, τ) entries of M_R are the only ones to be of order $\bar{m}_R$. Taking into account that $M_D = \mathrm{diag}\,(a, b, c)$ is diagonal, we find that the seesaw neutrino mass matrix of eq. (2.9) is

$$\mathcal{M}_\nu = \frac{1}{p^2 s + q^2 r - 2pqt + \epsilon^2 u (t^2 - rs)} \begin{pmatrix} \epsilon^2 a^2 (rs - t^2) & \epsilon ab (qt - ps) & \epsilon ac (pt - qr) \\ \epsilon ab (qt - ps) & b^2 (\epsilon^2 us - q^2) & bc (pq - \epsilon^2 ut) \\ \epsilon ac (pt - qr) & bc (pq - \epsilon^2 ut) & c^2 (\epsilon^2 ur - p^2) \end{pmatrix}. \tag{4.2}$$

Notice that only the (μ, μ), (τ, τ), and $(\mu, \tau) = (\tau, \mu)$ matrix elements of $\mathcal{M}_\nu$ are not ϵ-suppressed; still,

$$\left(\mathcal{M}_\nu\right)_{\mu\mu} \left(\mathcal{M}_\nu\right)_{\tau\tau} - \left(\mathcal{M}_\nu\right)_{\mu\tau} \left(\mathcal{M}_\nu\right)_{\tau\mu} = \frac{-\epsilon^2 b^2 c^2 u}{p^2 s + q^2 r - 2pqt + \epsilon^2 u \left(t^2 - rs\right)} \tag{4.3}$$

is suppressed by two powers of ϵ, and this fact is crucial in the following.

We shall perform an expansion in the small parameter ϵ and compute the subleading order in ϵ of the relevant quantities. This is consistent with our use of the seesaw formula of eq. (2.9); indeed, if one wanted to compute sub-subleading orders in the ϵ expansion one would first need a better version of the seesaw expansion of $\mathcal{M}_\nu$ [28].

JHEP07(2001)045

For the sake of simplicity we shall assume the matrix $\mathcal{M}_\nu$ of eq. (4.2) to be real.[6] This allows us to diagonalize it by simply looking for its eigenvalues and eigenvectors. Thus,

$$U^T \mathcal{M}_\nu U = \mathrm{diag}\left(\eta_1 m_1, \eta_2 m_2, \eta_3 m_3\right), \tag{4.4}$$

where the η_k may be either $+1$ or -1, and U is the neutrino mixing matrix as usual. If $\mathcal{M}_\nu$ was not real one would first have to diagonalize $\mathcal{M}_\nu \mathcal{M}_\nu^*$, and that would of course be quite more tedious. One of the eigenvalues (say, λ_0) of $\mathcal{M}_\nu$ is of order $(m_\nu^D)^2/m_R$, and the other two (we call them $\lambda_\pm$) are smaller, of order $(m_\nu^D)^2/\bar{m}_R$, because of eq. (4.3). The eigenvalues $\lambda_\pm$ fulfill $|\lambda_+| = |\lambda_-|$ to leading order in ϵ, which means that $\Delta m^2_\odot$ vanishes to that order. Explicitly, to subleading order in ϵ

$$\begin{aligned}
\lambda_0 &= \frac{-F^2}{p^2 s + q^2 r - 2pqt}, \\
\lambda_\pm &= \pm \frac{\epsilon abc}{F} + \frac{\epsilon^2 \left(a^2 b^4 q^2 s + a^2 c^4 p^2 r + 2a^2 b^2 c^2 pqt + b^2 c^2 F^2 u\right)}{2F^4},
\end{aligned} \tag{4.5}$$

where

$$F \equiv \sqrt{b^2 q^2 + c^2 p^2}\,. \tag{4.6}$$

It follows that

$$\begin{aligned}
\frac{\Delta m^2_\odot}{\Delta m^2_{\mathrm{atm}}} &\simeq \frac{\left|\lambda_+^2 - \lambda_-^2\right|}{\lambda_0^2} \\
&\simeq \left| \frac{2\epsilon^3 abc \left(p^2 s + q^2 r - 2pqt\right)^2 \left(a^2 b^4 q^2 s + a^2 c^4 p^2 r + 2a^2 b^2 c^2 pqt + b^2 c^2 F^2 u\right)}{F^9} \right|
\end{aligned} \tag{4.7}$$

is of order ϵ^3. Maximal solar neutrino mixing is allowed at 90% CL in the LOW solution with $\Delta m^2_\odot \sim 10^{-7}\,\mathrm{eV}^2$, giving the estimate $\epsilon^3 \sim 10^{-4}$. In this case ϵ is small enough for a sensible expansion. Approximate maximal mixing is also possible in the LMA MSW solution, but then only at 99% CL [4]; in that case $\epsilon^3 \sim 10^{-2}$ is rather large and the two scales m_R and $\bar{m}_R$ are not so clearly separated.

To subleading order in ϵ, the normalized eigenvector corresponding to the eigenvalue λ_0 is

$$\begin{pmatrix} \epsilon a \left[\left(b^2 q^2 - c^2 p^2\right) t + pq\left(c^2 r - b^2 s\right)\right]/F^3 \\ -bq/F \\ cp/F \end{pmatrix}, \tag{4.8}$$

and the ones corresponding to the eigenvalues $\lambda_\pm$ are

$$\begin{pmatrix} \frac{1}{\sqrt{2}}\left[1 \pm \epsilon\left(\frac{abcpqt}{2F^3} + \frac{ac^3 p^2 r}{4bF^3} + \frac{ab^3 q^2 s}{4cF^3} - \frac{bcu}{4aF}\right)\right] \\ \mp \frac{cp}{\sqrt{2}F}\left[1 \pm \mathrm{O}(\epsilon)\right] \\ \mp \frac{bq}{\sqrt{2}F}\left[1 \pm \mathrm{O}(\epsilon)\right] \end{pmatrix}, \tag{4.9}$$

[6] By imposing the CP symmetry of eq. (3.18), and with the help of a phase transformation, the imaginary matrix M_R can be made real while keeping M_D real. Then $\mathcal{M}_\nu$ will be real too.

respectively. It follows that, to leading order, the mixing matrix U is

$$U = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ \sin\psi/\sqrt{2} & -\sin\psi/\sqrt{2} & -\cos\psi \\ \cos\psi/\sqrt{2} & -\cos\psi/\sqrt{2} & \sin\psi \end{pmatrix} \qquad \text{with } \sin\psi \equiv \frac{cp}{F}, \ \cos\psi \equiv \frac{bq}{F}. \tag{4.10}$$

Thus, $U_{e3} = 0$ and maximal solar mixing hold at leading order. At subleading order

$$\begin{aligned} U_{e3} &= \epsilon \, \frac{a\left[\left(b^2q^2 - c^2p^2\right)t + pq\left(c^2r - b^2s\right)\right]}{F^3}, \\ \sin^2 2\theta &= 1 - \epsilon^2 \left(\frac{abcpqt}{F^3} + \frac{ac^3p^2r}{2bF^3} + \frac{ab^3q^2s}{2cF^3} - \frac{bcu}{2aF}\right)^2. \end{aligned} \tag{4.11}$$

It is worth emphasizing that $\Delta m^2_\odot/\Delta m^2_{\mathrm{atm}}$ is of order ϵ^3 while $1 - \sin^2 2\theta$ is proportional to ϵ^2 only. This means that, for solutions of the solar neutrino problem with a higher value of $\Delta m^2_\odot/\Delta m^2_{\mathrm{atm}}$, such as the LMA MSW solution, we may allow $\sin^2 2\theta$ to be further away from unity. This gives us extra room in the fitting of the experimental data within the context of our model.

We also remark that some models based on $\bar{L}$ symmetry [14] predict a massless neutrino ($m_3 = 0$) which has zero component along the ν_e direction ($U_{e3} = 0$). The present model, on the contrary, allows for U_{e3} to differ from 0 substantially (as it is only of order ϵ) and, moreover, it predicts m_3 to be much larger than m_1 and m_2, instead of $m_3 = 0$. Note that our model displays $m_2 - m_1 \ll (m_1 + m_2)/2$, contrary to the orthodox hierarchical pattern of masses, where $m_1 \ll m_2$.

5. Bimaximal mixing

We may combine the assumptions of sections 3 and 4 to obtain a scheme with *natural* bimaximal mixing. In that scheme $M_D = \mathrm{diag}\,(a, b, b)$ as in eq. (3.4) and

$$M_R = \begin{pmatrix} u & p/\epsilon & p/\epsilon \\ p/\epsilon & r & t \\ p/\epsilon & t & r \end{pmatrix}, \tag{5.1}$$

with ϵ much smaller than 1, in analogy to eq. (4.1). Then, assuming once again M_D and M_R to be real for the sake of simplification (see footnote 6), we obtain

$$\begin{aligned} \lambda_0 &= \frac{b^2}{t - r}, \\ \lambda_\pm &= \pm \frac{\epsilon ab}{\sqrt{2}p} + \frac{\epsilon^2\left[a^2(r+t) + b^2u\right]}{4p^2}, \end{aligned} \tag{5.2}$$

and

$$\sin^2 2\theta = 1 - \frac{\epsilon^2}{8p^2}\left[\frac{a}{b}(r+t) - \frac{b}{a}u\right]^2. \tag{5.3}$$

Maximal atmospheric neutrino mixing and $U_{e3} = 0$ are exact results in this case. Indeed, the full 6×6 neutrino mass matrix

$$\mathcal{M}_{D+M} = \begin{pmatrix} 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & b & 0 \\ 0 & 0 & 0 & 0 & 0 & b \\ a & 0 & 0 & u & p/\epsilon & p/\epsilon \\ 0 & b & 0 & p/\epsilon & r & t \\ 0 & 0 & b & p/\epsilon & t & r \end{pmatrix} \tag{5.4}$$

has an eigenvalue

$$(t-r)\,\frac{\left(\sqrt{1+\sigma^2}-1\right)}{2} \simeq \frac{b^2}{t-r}\,, \tag{5.5}$$

where

$$\sigma \equiv \frac{2b}{r-t}\,. \tag{5.6}$$

The above eigenvalue is seesaw-suppressed and its absolute value corresponds to m_3. To the exact eigenvalue (5.5) corresponds the exact eigenvector of $\mathcal{M}_{D+M}$

$$\frac{1}{2\left(1+\sigma^2\right)^{1/4}} \begin{pmatrix} 0 \\ \sigma/\eta \\ -\sigma/\eta \\ 0 \\ -\eta \\ \eta \end{pmatrix}, \qquad \text{with } \eta \equiv \sqrt{\sqrt{1+\sigma^2}-1} \simeq \frac{\sigma}{\sqrt{2}}\,. \tag{5.7}$$

Note that both the eigenvalue and the eigenvector depend only on b and on $r-t$; in particular, they depend neither on a nor on p.

6. Conclusions

In this paper we have considered the extension of the SM with three right-handed neutrino singlets with large Majorana mass terms responsible for the seesaw mechanism. Upon this standard scenario we have imposed the separate conservation of the family lepton numbers L_e, L_μ, and L_τ in the Yukawa couplings, such that these lepton numbers are softly broken solely by the Majorana mass terms. In this way, at tree level the charged-lepton mass matrix is automatically diagonal and lepton mixing originates exclusively in the Majorana mass matrix M_R.

This makes it relatively easy to impose a discrete symmetry which enforces $U_{e3} = 0$ and maximal atmospheric neutrino mixing; for this we need three Higgs doublets. An alternative model, with maximal solar mixing, is obtained when there are two different scales in M_R such that, at the higher scale, the individual lepton numbers L_e, L_μ, and L_τ are broken down to $\bar{L} = L_e - L_\mu - L_\tau$; whereas at the lower scale $\bar{L}$ is also softly broken. In this case one Higgs doublet suffices. The two models can easily be combined if one wants to obtain bimaximal mixing.

JHEP07(2001)045

In the model with maximal atmospheric mixing the ratio $\Delta m^2_\odot/\Delta m^2_{\rm atm}$ has to be fitted by means of a finetuning. In the model with maximal solar mixing there is a relationship between the deviation of θ from 45° and the ratio of the mass-squared differences, namely $1 - \sin^2 2\theta \sim \epsilon^2$ and $\Delta m^2_\odot/\Delta m^2_{\rm atm} \sim \epsilon^3$, where ϵ is the ratio of the two scales in M_R.

The mechanisms for maximal neutrino mixing discussed in this paper are extremely simple and require only a minimal extension of the SM with right-handed singlets ν_R. Apart from the possibility of obtaining maximal neutrino mixing in a natural way, the violation of L_e, L_μ, and L_τ exclusively by the large Majorana masses of the ν_R constitutes in itself an interesting scenario, where deviations from the conservation of the family lepton numbers are calculable, finite, and controlled by M_R only.

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JHEP07(2001)045